MPIDR-NES Training Programme

Moscow, New Economic School,14th January - 1st February 2013

Population and Health

Лекция 3: Таблица дожития и причины смерти.
Lecture 3: Life table and causes of death.

MAX PLANCK INSTITUTE FOR DEMOGRAPHIC RESEARCH MAX-PLANCK-INSTITUT FÜR DEMOGRAFISCHE FORSCHUNG

РЭШ Российская экономическая школа

Single and multiple decrement processes

So far, we have been considering situations in which people were exposed to risk of only one absorbing event (death). Process of the population reduction with age could be named a single-decrement process.

People can be also exposed to *competing risks* of several events simultaneously. If these events are excluding, then the process is a *multiple decrement* one.

Examples:

- married people are exposed to risks of divorce, widowhood, and death;

- people are exposed to risk of migration to various places;

- people are exposed to risk of acquiring various diseases;

- ill people are exposed to risks of cure or death;

- *people are exposed to risk of death from different medical causes*.

Death rates and deaths in a multiple decrement life table

Let m_i^j be a life table death rate for cause *j* and age group *i.*

$$m_i^j = \frac{d_i^j}{L_i}$$

Let M_i^j be an empirical death rate for cause *j* and age group *i.*

$$M_i^j = \frac{D_i^j}{E_i} = r_i^j \cdot \frac{D_i}{E_i} = r_i^j \cdot M_i$$

where r_i^j is a share of deaths from cause *j* among all deaths at age *i*.
Taking in account that the empirical and the life table death rates are equal we obtain:

$$m_i^j = M_i^j = r_i^j \cdot M_i = r_i^j \cdot m_i,$$

$$d_i^j = r_i^j \cdot d_i,$$

$$d_i = \sum_j d_i^j$$

Age-specific probabilities of death and probabilities of eventually dying from cause *j*

Probability of leaving the life table due to cause *j* is

$$q_i^j = \frac{d_i^j}{l_{x_i}}$$

Its value shows probability of dying within age interval $[x_i, x_{i+1})$ from cause *j* out of those who survived *all causes* of mortality to age x_i

Among survivors to age x_i, it is possible to distinguish individuals, who will eventually die from cause *j* after *age* x_i :

$$l_{x_i}^j = \sum_{k=i}^{w} d_k^j$$

For every age x_i :

$$l_{x_i} = \sum_j l_{x_i}^j$$

Share of such individuals among all survivors to age x_i, is

$$\theta_{x_i}^j = \frac{l_{x_i}^j}{l_{x_i}}$$

Total share of those eventually dying from cause *j* is

$$\theta_0^j = \frac{l_0^j}{l_0}$$

Mean ages at death for causes of death

If one assumes that within each elementary age interval [x_i, x_{i+1}) average age of those dying is the same for every cause of death, then mean age at death for any cause *j* can be expressed as

$$e_{x_i}^j = \frac{1}{l_{x_i}^j} \sum_{i=0}^{w} \bar{x}_i \, d_i^j$$

The overall life expectancy at age x_i is a weighted average of cause-specific mean ages at death

$$e_{x_i} = \sum_j \theta_i^j e_{x_i}^j$$

Calculation of a multiple decrement LT can be performed in the following sequences

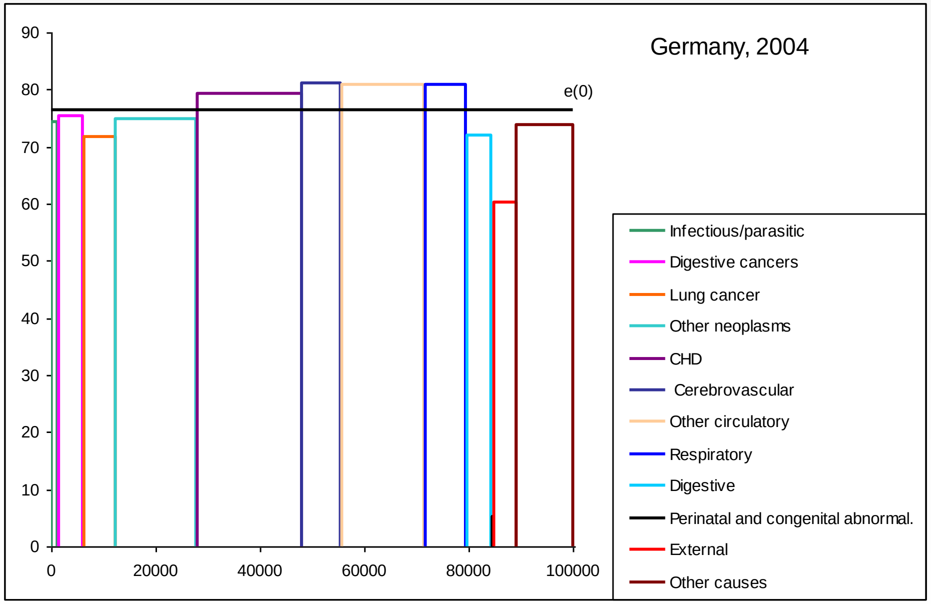
$$D_i^j \rightarrow r_i^j, d_i \rightarrow d_i^j \rightarrow l_{x_i}^j$$

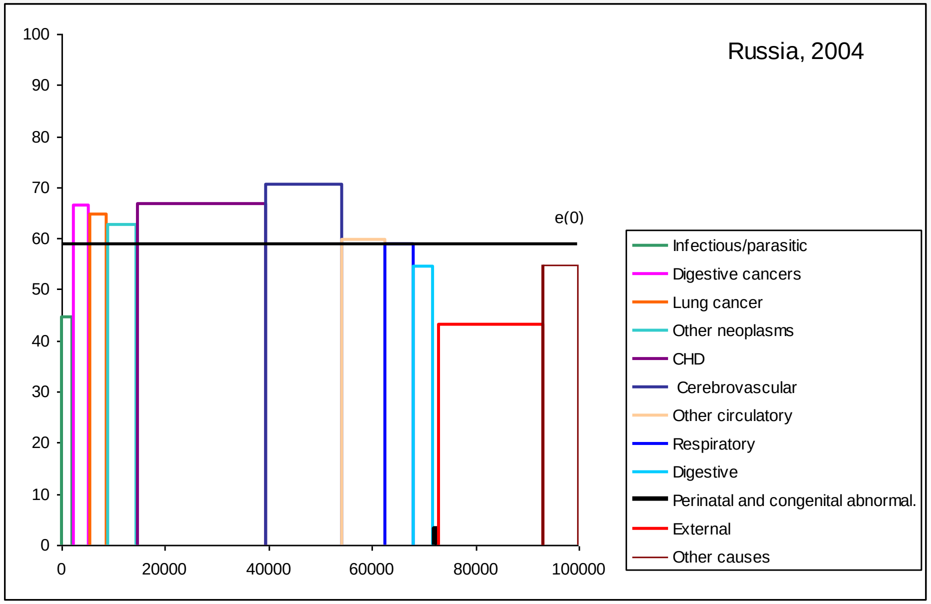
$$d_i^j, \bar{x}_i \rightarrow e_{x_i}^j$$

MultipleDecrementLifeTable.xls

Graphic presentation of multiple decrement LT.
A Germany-Russia comparison of cause-of-death patterns.

Each bar corresponds to one cause of death. Its width presents share of the LT cohort eventually dying from the cause. Its height presents mean age at death for the cause.





Graphic-presentation-MDLT-FUL.xls

Cause-elimination in a LT (an associated single-decrement LT)

How many years of life can be gained if cause of death *j* is eliminated? The associated single decrement LT can tell about it.

Let *k* designate all causes of death other than *j*. If cause *j* is eliminated, cause *k* is the only one operating in the cohort. Thus, the probability of surviving from age x_i to age x_{i+1} is

$$p_i^k = e^{-\int_{x_i}^{x_{i+1}} \mu^k(t)dt} = e^{-r_i^k \int_{x_i}^{x_{i+1}} \mu(t)dt} = (p_i)^{r_i^k} \qquad r_i^k = \frac{M_i^k}{M_i} = \frac{D_i^k}{D_i}$$

Once p_i^k are known, they allow calculating the whole LT according to the standard procedure

$$p_i^k \to q_i^k \to l_i^k \to L_i^k \to T_{x_i}^k \to e_{x_i}^k$$

Values of a_i^k are needed in this calculation. The a_i^k can be set equal to a_i from the master LT. For better precision, a special procedure can be applied (as described in the next slide).

Incorporating causes of death: cause-elimination in LT (an associated single-decrement LT)

For 5-year age groups the parabolic interpolation on d_i^k over three neighboring age groups yields:

$$a_i^k = \frac{-(1/24)d_{i-1}^k + (1/2)d_i^k + (1/24)d_{i+1}^k}{d_i^k}$$

For ages 0, 1-4, and 5-9 the following formula can be used

$$a_i^k = n_i + r_i^k \frac{q_i}{q_i^k}(a_i - 1)$$

Source: Preston, Heuveline & Guillot, 2001, p.84.

AssociatedSingleDecrementLT.xls

Effects of elimination of selected causes for Germany and Russia.

	Germany	Gain in LEB	Russia	Gain in LEB
Initial LEB	76.5	0.0	58.9	0.0
Eliminated cause				
Cancers of the digestive system	77.1	0.6	59.2	0.3
Lung cancer	77.4	0.9	59.3	0.4
IHD	79.0	2.5	62.3	3.4
Cerebrovascular	77.3	0.8	60.6	1.7
All CVD	84.1	7.6	70.6	11.7
External	77.6	1.1	64.2	5.3

Public health evaluation of causes of death: YLL

One way of evaluation of relative importance of different causes of death is to compare their weights in the total number of deaths. As we saw, another way is to estimate gains of life expectancy produced by cause-eliminations (associated single decrement life tables).

The WHO's **years of life lost** is one more useful criterion for evaluation of causes of death. YLL is equal to person-years of length of life lost in comparison to a target good practice ("standard") population. It shows how much time more could be lived if the mortality difference from the standard population was eliminated.

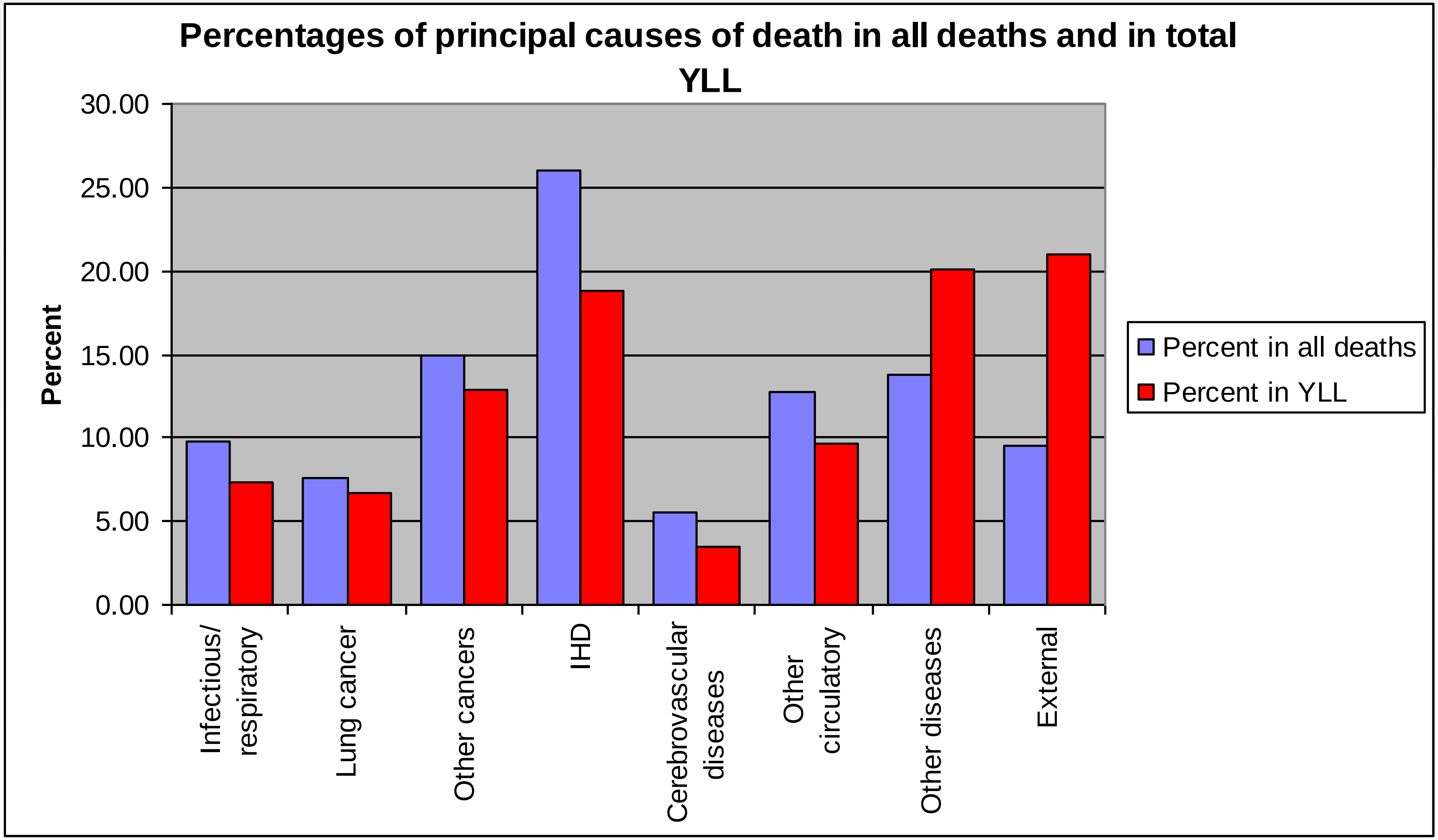
$$YLL_i^j = D_i^j(e_{\bar{x}_i}^s - \bar{x}_i)$$ age-specific YLL

$$YLL^j = \sum_{i=0}^{w} D_i^j(e_{\bar{x}_i}^s - \bar{x}_i)$$ total YLL

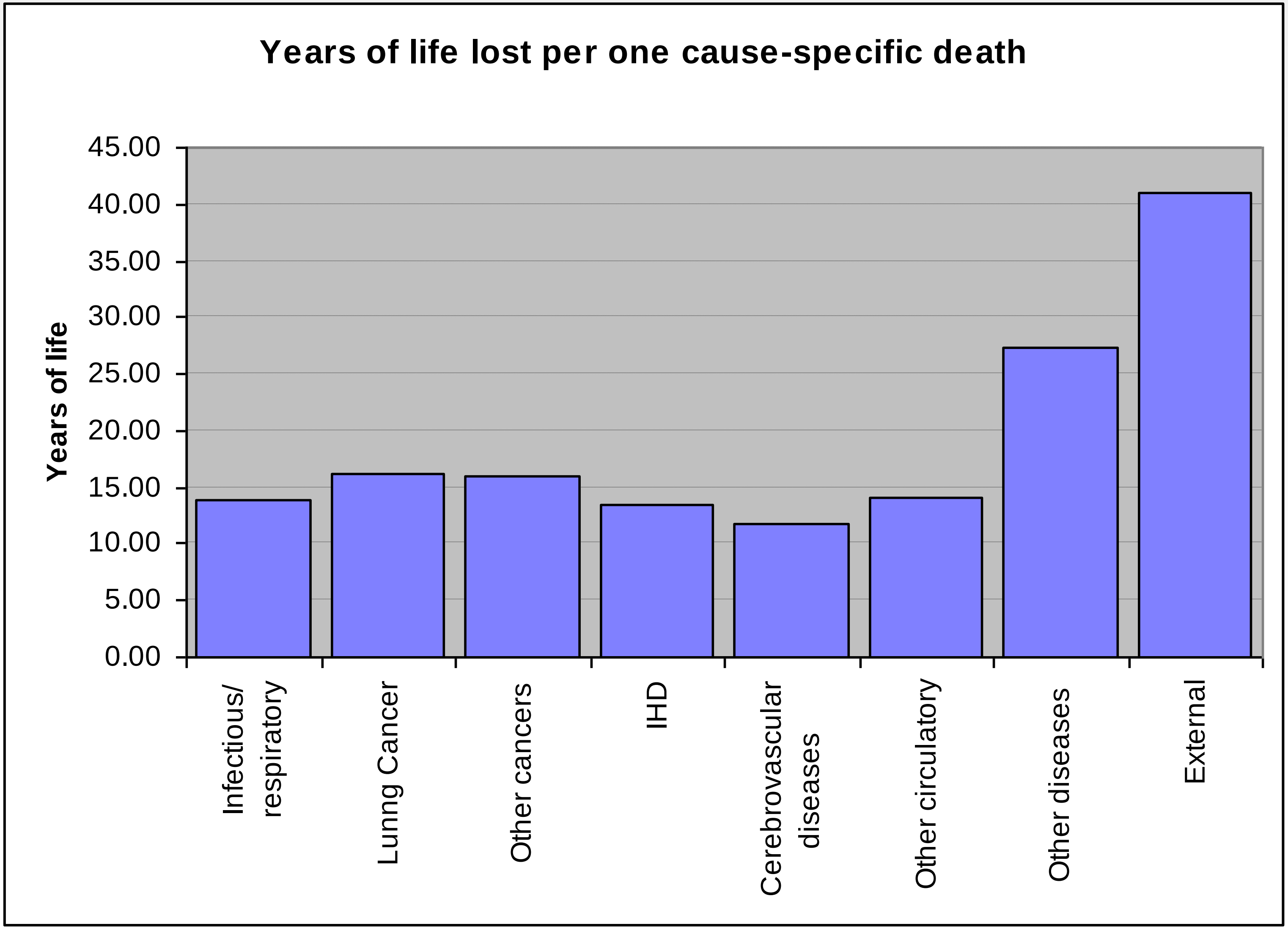
Relative YLL measures can be calculated by dividing the absolute YLL measures by population size (lost life years per thousand residents), by number of deaths (lost life years per one death), and by number of deaths from corresponding cause (lost life years per one cause-specific death).

YLL-US-males.xls

Public health evaluation of causes of death: YLL



Public health evaluation of causes of death: YLL



Life table losses of length of life: e-dagger

YLL is calculated directly from observed deaths and depends on population age structure and on choice of the standard life table.

N.Keyfitz (1977) followed by Vaupel and Canudas-Romo (2003) proposed a life table quantity named e-dagger for measuring losses of life due to death. N.Keyfitz: “everybody dies prematurely” since every death “deprives the person involved of the reminder of his expectation of life” (Keyfitz, 1977: 61-68). e-dagger is based on counting lifetime lost in comparison with expected average length of life in the same life table cohort.

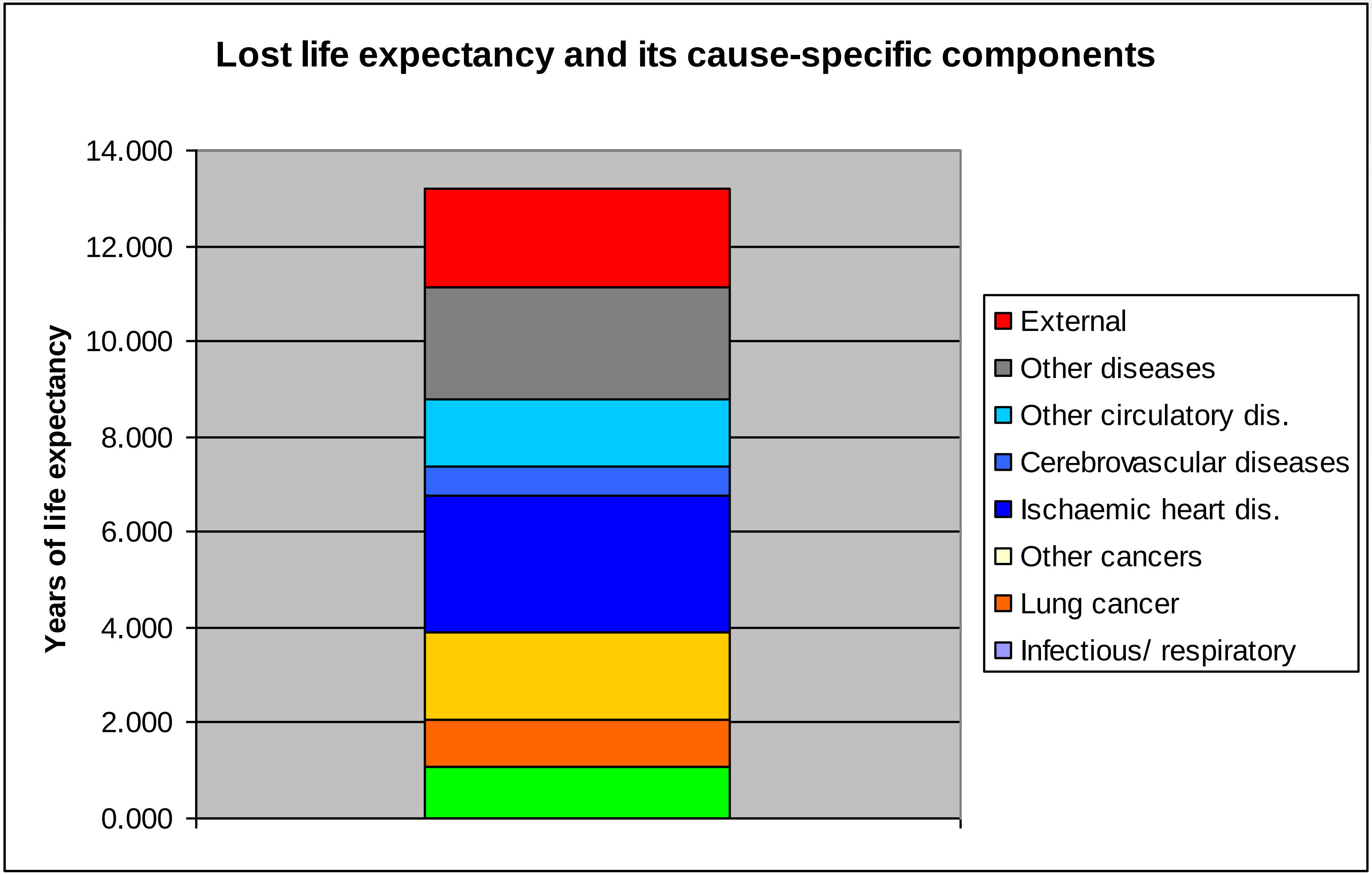
age-specific component $$d_i(e_{x_i} + e_{x_{i+1}}) \cdot \frac{1}{2}$$

total e-dagger $$e^{\dagger}_{x_i} = \frac{1}{l_{x_i}} \sum_{y=x_i}^{\omega-1} d_y \bar{e}_y = \frac{1}{2l_{x_i}} \sum_{y=x}^{\omega-1} d_y \left(e_y + e_{y+n}\right)$$

E-dagger shows how many years of life are lost due to an average death relative to the average survival. E-dagger reflects amount of inter-individual disparity in length of life in the life table cohort.

E-dagger-US-males.xls

Life table losses of length of life: e-dagger



References

Chiang C. L. (1984). *The Life Table and its Applications*. Robert E Krieger Publ Co.: Malabar (FL)

Preston, S.H., Heuveline, P., Guillot, M. (2001). Demography. Measuring and modeling population processes. Malden, Oxford: Blackwell Publishing.

Vaupel, J. W.; and V. Canudas Romo. 2003. “Decomposing change in life expectancy: a bouquet of formulas in honor of Nathan Keyfitz´s 90th birthday.” Demography 40(2):201–216.

http://www.who.int/healthinfo/global_burden_disease/

http://www.who.int/healthinfo/global_burden_disease/metrics_daly/en/

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Population and Health. *MPIDR-NES Training Programme 2013*